SOLUTION OF SHORT QUESTIONS

Short Questions

Write the short answers of the following Questions:

Q.1: What is a scalar? Give examples.

Ans. A scalar is a quantity having magnitude only but no direction.

Examples: Length, Mass, Time, Volume, etc.

Q.2: What is a vector? Give examples.

Ans. A vector is a quantity having both magnitude and direction.

Examples: Force, Velocity, Acceleration, etc.

Q.3: What is a unit vector?

Ans. A vector whose magnitude is unity is called a unit vector.

Q.4: Find the formula for magnitude of the vector $\vec{r} = xi + yj + zk$.

Sol. Magnitude $|\vec{r}| = \sqrt{x^2 + y^2 + z^2}$ **(IA-2021)**

Q.5: Find the magnitude of the vector $-2i - 4j + 3k$.

Sol. Let $\vec{a} = -2i - 4j + 3k$ **(IIA-2020), (IIA-2021)**

$$|\vec{a}| = \sqrt{(-2)^2 + (-4)^2 + (3)^2} = \sqrt{4 + 16 + 9} = \boxed{\sqrt{29}}$$

Q.6: What are parallel vectors? **(IIA-2016), (IA-2019)**

Sol. Two vectors $\vec{a}$ and $\vec{b}$ are parallel if there exist a non-zero $k \varepsilon R$, such that $\vec{a} = k\vec{b}$

Q.7: Find 'α', so that $|\alpha i + (\alpha + 1)j + 2k| = 3$ **(IIA-2018), (IA-2022)**

Sol. $|\alpha i + (\alpha + 1)j + 2k| = 3$

$$\Rightarrow \sqrt{(\alpha)^2 + (\alpha+1)^2 + (2)^2} = 3$$

$$\Rightarrow \sqrt{\alpha^2 + \alpha^2 + 2\alpha + 1 + 4} = 3$$

$$\Rightarrow \sqrt{2\alpha^2 + 2\alpha + 5} = 3$$

Squaring both sides, we get

$$2\alpha^2 + 2\alpha + 5 = 9$$

$$2\alpha^2 + 2\alpha + 5 - 9 = 0$$

SOLUTION OF SHORT QUESTIONS

$2\alpha^2 + 2\alpha - 4 = 0$

$2\alpha^2 + 4\alpha - 2\alpha - 4 = 0$ {By Factorization}

$2\alpha(\alpha + 2) - 2(\alpha + 2) = 0$

$\Rightarrow (\alpha + 2)(2\alpha - 2) = 0$

Either $\alpha + 2 = 0$ or $2\alpha - 2 = 0$

$\boxed{\alpha = -2}$ | $2\alpha = 2 \Rightarrow \boxed{\alpha = 1}$

Q.8: **If $\cos\alpha$, $\cos\beta$, $\cos\gamma$ are direction cosines of a vector $\vec{r} = xi + yj + zk$, then show that $\cos^2\alpha + \cos^2\beta, \cos^2\gamma = 1$.**

Sol. As, $\vec{r} = xi + yj + zk \Rightarrow |\vec{r}| = \sqrt{x^2 + y^2 + z^2}$ **(IA-2018)**

Direction cosines of $\vec{r}$ are:

$$\cos\alpha = \frac{x}{\sqrt{x^2 + y^2 + z^2}} \Rightarrow \cos^2\alpha = \frac{x^2}{x^2 + y^2 + z^2} \rightarrow (i)$$

$$\cos\beta = \frac{y}{\sqrt{x^2 + y^2 + z^2}} \Rightarrow \cos^2\beta = \frac{y^2}{x^2 + y^2 + z^2} \rightarrow (ii)$$

$$\cos\gamma = \frac{z}{\sqrt{x^2 + y^2 + z^2}} \Rightarrow \cos^2\gamma = \frac{y^2}{x^2 + y^2 + z^2} \rightarrow (iii)$$

Adding eq.(i), eq.(ii) & eq.(iii)

$\text{L.H.S.} = \cos^2\alpha + \cos^2\beta + \cos^2\gamma$

$$= \frac{x^2}{x^2 + y^2 + z^2} + \frac{y^2}{x^2 + y^2 + z^2} + \frac{z^2}{x^2 + y^2 + z^2}$$

$$= \frac{x^2 + y^2 + z^2}{x^2 + y^2 + z^2} = 1 = \text{R.H.S.}$$ **Proved**

Q.9: **Find the unit vector along the vector $4i - 3j - 5k$.**

(IIA-2016), (IA-2017), (IIA-2019), (IIA-2021)

Sol. Let $\vec{a} = 4i - 3j - 5k$

$|\vec{a}| = \sqrt{(4)^2 + (-3)^2 + (-5)^2}$

$= \sqrt{16 + 9 + 25} = \sqrt{50} = \sqrt{25 \times 2} = 5\sqrt{2}$

SOLUTION OF SHORT QUESTIONS

Unit vector $= \hat{a} = \frac{\vec{a}}{|\vec{a}|} = \boxed{\frac{4i-3j-5k}{5\sqrt{2}}}$

Q.10: Find the unit vector parallel to the sum of the vectors: $\vec{a} = [2, 4, -5]$, $\vec{b} = [1, 2, 3]$ (IA-2016)

Sol. Let $\vec{v}$ = sum of the vectors $\vec{a}$ & $\vec{b} = \vec{a} + \vec{b}$

$= [2,4,-5] + [1,2,3] = [3, 6, -2] = 3i + 6j - 2k$

$|\vec{v}| = \sqrt{(3)^2 + (6)^2 + (-2)^2} = \sqrt{9+36+4} = \sqrt{49} = 7$

unit vector $= |\hat{v}| = \frac{\vec{v}}{|\vec{v}|} = \boxed{\frac{3i+6j-2k}{7}}$

Q.11: Given the vectors:

$\vec{a} = 3i - 2j + k$, $\vec{b} = 2i - 4j - 3k$, $\vec{c} = -i + 2j + 2k$,

Find $\vec{a} + \vec{b} + \vec{c}$ (IA-2019), (IIA-2020)

Sol. $\vec{a} + \vec{b} + \vec{c}$

$= 3i - 2j + k + 2i - 4j - 3k - i + 2j + 2k = \boxed{4i - 4j + 0k}$

Q.12: Given the vectors $\vec{a} = 3i + j - k$ and $\vec{b} = 2i + j - k$, find magnitude of $3\vec{a} - \vec{b}$. (IA-2017), (IIA-2017)

Sol. $3\vec{a} - \vec{b} = 3(3i + j - k) - (2i + j - k)$

$= 9i + 3j - 3k - 2i - j + k = 7i + 2j - 2k$

$|3\vec{a} - \vec{b}| = \sqrt{(7)^2 + (2)^2 + (-2)^2} = \sqrt{49+4+4} = \boxed{\sqrt{57}}$

Q.13: Find a vector whose magnitude is 2 and is parallel to 5i + 3j + 2k. (IA-2017), (IA-2018), (IA-2022)

Sol. Let $\vec{a}$ be a require vector, so $|\vec{a}| = 2$

& Let $\vec{b} = 5i + 3j + 2k$

$|\vec{b}| = \sqrt{(5)^2 + (3)^2 + (2)^2} = \sqrt{25+9+4} = \sqrt{38}$

As $\vec{a}$ and $\vec{b}$ are parallel vectors:

So $\hat{a} = \hat{b}$

SOLUTION OF SHORT QUESTIONS

$$\frac{\vec{a}}{|\vec{a}|} = \frac{\vec{b}}{|\vec{b}|}$$

$$\frac{\vec{a}}{2} = \frac{5i+3j+2k}{\sqrt{38}}$$

$$\vec{a} = \frac{2(5i+3j+2k)}{\sqrt{38}} \Rightarrow \vec{a} = \boxed{\frac{10i+6j+4k}{\sqrt{38}}}$$

Q.14: Define scalar product of two vectors.

Sol. The scalar product of two vectors $\vec{a}$ & $\vec{b}$ is denoted by $\vec{a}\,.\,\vec{b}$ and defined as $\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta$.

Q.15: Find $\vec{a}.\vec{b}$ if $\vec{a} = i + 2j + 2k$, $\vec{b} = 3i - 2j - 4k$.

(IA-2021)

Sol. $\vec{a}.\vec{b} = (i + 2j + 2k) . (3i - 2j - 4k)$

$= (1)(3) + (2)(-2) + (2)(-4) = 3 - 4 - 8 = \boxed{-9}$

Q.16: Find $(\vec{a}+\vec{b}).(\vec{a}-\vec{b})$ if $\vec{a} = 2i + 2j + 3k$,

$\vec{b} = 2i - j + k$. **(IA-2016), (IA-2017), (IIA-2020)**

Sol.

$\vec{a}+\vec{b}$	$\vec{a}-\vec{b}$
$= (2i + 2j + 3k) + (2i - j + k)$	$= (2i + 2j + 3k) - (2i - j + k)$
$= 2i + 2j + 3k + 2i - j + k$	$= 2i + 2j + 3k - 2i + j - k$
$= 4i + j + 4k$	$= 3j + 2k$

$(\vec{a}+\vec{b}).(\vec{a}-\vec{b}) = (4i + j + 4k) . (3j + 2k)$

$= (4)(0) + (1)(3) + (4)(2) = 0 + 3 + 8 = \boxed{11}$

Q.17: Define vector product. **(IIA-2018)**

Ans. The vector product of two vectors $\vec{a}$ & $\vec{b}$ is denoted by $\vec{a}\times\vec{b}$ and is defined as $\vec{a}\times\vec{b} = |\vec{a}||\vec{b}|\sin\theta\,\hat{n}$.

Q.18: If $\vec{a} = 2i + 3j + 4k$, $\vec{b} = i - j + k$ find $|\vec{a}\times\vec{b}|$

SOLUTION OF SHORT QUESTIONS

Sol. $\vec{a}\times\vec{b} = \begin{vmatrix} i & j & k \\ 2 & 3 & 4 \\ 1 & -1 & 1 \end{vmatrix} = i\begin{vmatrix} 3 & 4 \\ -1 & 1 \end{vmatrix} - j\begin{vmatrix} 2 & 4 \\ 1 & 1 \end{vmatrix} + k\begin{vmatrix} 2 & 3 \\ 1 & -1 \end{vmatrix}$

$= i(3-(-4)) - j(2-4) + k(-2-3)$

$= i(3+4) - j(-2) + k(-5) = 7i + 2j - 5k$

$|\vec{a}\times\vec{b}| = \sqrt{(7)^2+(2)^2+(-5)^2} = \sqrt{49+4+25} = \boxed{\sqrt{78}}$

Q.19: Find the area of parallelogram with adjacent sides, $\vec{a} = 7i - j + k$ & $\vec{b} = 2j - 3k$ (IA-2019), (IIA-2020)

Sol. $\vec{a}\times\vec{b} = \begin{vmatrix} i & j & k \\ 7 & -1 & 1 \\ 0 & 2 & -3 \end{vmatrix} = i\begin{vmatrix} -1 & 1 \\ 2 & -3 \end{vmatrix} - j\begin{vmatrix} 7 & 1 \\ 0 & -3 \end{vmatrix} + k\begin{vmatrix} 7 & -1 \\ 0 & 2 \end{vmatrix}$

$= i(3-2) - j(-21-0) + k(14+0)$

$= i(1) - j(-21) + k(14) = i + 21j + 14k$

$|\vec{a}\times\vec{b}| = \sqrt{(1)^2+(21)^2+(14)^2} = \sqrt{1+441+196} = \sqrt{638}$

Area of parallelogram $= |\vec{a}\times\vec{b}| = \boxed{\sqrt{638}\text{ sq. unit}}$

Q.20: For what value of λ, the vectors $2i - j + 2k$ and $3i + 2\lambda j$ are perpendicular. (IIA-2016), (IIA-2017)

Sol. Let $\vec{a} = 2i - j + 2k$ & $\vec{b} = 3i + 2\lambda j$

As given vectors are perpendicular.

So, $\vec{a}.\vec{b} = 0$

$\Rightarrow (2i - j + 2k).(3i + 2\lambda j) = 0$

$\Rightarrow (2)(3) + (-1)(2\lambda) + (2)(0) = 0$

$\Rightarrow 6 - 2\lambda + 0 = 0$

$\Rightarrow -2\lambda = -6 \Rightarrow \lambda = \dfrac{-6}{-2} \Rightarrow \boxed{\lambda = 3}$

Q.21: Under what conditions does the relation $\vec{a}.\vec{b} = |\vec{a}||\vec{b}|$ hold.

(IA-2016), (IIA-2018)

SOLUTION OF SHORT QUESTIONS

Sol. As $\vec{a}.\vec{b} = |\vec{a}||\vec{b}|$

$\Rightarrow \quad |\vec{a}||\vec{b}|\cos\theta = |\vec{a}||\vec{b}| \qquad \because \left\{ \begin{array}{l} \text{By Definition} \\ \vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta \end{array} \right\}$

$\Rightarrow \quad \cos\theta = \dfrac{|\vec{a}||\vec{b}|}{|\vec{a}||\vec{b}|} \qquad \Rightarrow \qquad \cos\theta = 1$

$\Rightarrow \quad \theta = \cos^{-1}(1) \Rightarrow \quad \boxed{\theta = 0^\circ}$

Q.22: Find the scalars x and y such that $x(i+2j)+y(3i+4j)=7i+9j$

Sol. $x(i+2j)+y(3i+4j)=7i+9j$ **(IIA-2020)**

$xi+2xj+3yi+4yj=7i+9j$

$(x+3y)i+(2x+4y)j=7i+9j$

Comparing coefficient of i and j, we have.

$x+3y=7 \rightarrow (i)$ | $2x+4y=9 \rightarrow (ii)$

Multiplying eq.(i) by 2: $2x+6y=14 \rightarrow (iii)$

Subtracting eq.(iii) & eq.(ii)

$$\begin{array}{r} 2x+6y=14 \\ -2x \pm 4y = -9 \\ \hline 2y=5 \end{array} \Rightarrow \boxed{y = \tfrac{5}{2}}$$

Put $y = \frac{5}{2}$ in eq.(i)

$x + 3\left(\frac{5}{2}\right) = 7$

$x = 7 - \dfrac{15}{2} = \dfrac{14-15}{2} = \boxed{-\dfrac{1}{2}}$

Q.23: Prove that if $\vec{a} = i + 3j - 2k$, and $\vec{b} = i - j - k$, then $\vec{a}$ and $\vec{b}$ are perpendicular to each other.

(IA-2018)

Sol. $\vec{a}.\vec{b} = (i+3j-2k).(i-j-k)$

$= (1)(1)+(3)(-1)+(-2)(-1) = 1-3+2 = \boxed{0}$

Hence $\vec{a}$ and $\vec{b}$ are perpendicular vectors.